Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2024

Centre Number

Candidate Number

Further Mathematics

Assessment Unit AS 1

assessing

Pure Mathematics

[SFM11]

SFM11

MONDAY 13 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all eight** questions in the spaces provided.
Do not write outside the boxed area on each page or on blank pages.
Complete in black ink only. **Do not write with a gel pen.**
Questions which require drawing or sketching should be completed using an HB pencil.
Show clearly the full development of your answers. **Answers without working may not gain full credit.**
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

1 Matrices **M** and **I** are defined by

$$\mathbf{M} = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} \quad \text{and} \quad \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

(i) Show that $\mathbf{M}^2 = 5\mathbf{M} - 6\mathbf{I}$ [4]

(ii) Hence, or otherwise, express the matrix $\mathbf{M}^4$ in the form $a\mathbf{M} + b\mathbf{I}$, where a and b are integers. [4]

2 The roots of the quadratic equation

$$3x^2 - 2x + 4 = 0$$

are α and β.

(i) Write down the values of $(\alpha + \beta)$ and $\alpha\beta$. [3]

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(ii) Find a quadratic equation with integer coefficients whose roots are

$\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ [6]

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3 The matrix $\mathbf{P} = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$

(i) Describe fully the single transformation represented by **P** [3]

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The matrix $\mathbf{Q} = \begin{pmatrix} 1 & -1 \\ 0 & 3 \end{pmatrix}$

The matrix **S** represents the combined effect of the transformation represented by **P** followed by the transformation represented by **Q**

(ii) Find the matrix **S** [2]

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A region R is mapped to a new region T under the transformation represented by **S**

(iii) If the area of T is 60 cm^2, find the area of R. [3]

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4 **(a)** The complex numbers z_1 and z_2 are given by

$$z_1 = 2 + 2\mathrm{i} \quad \text{and} \quad z_2 = \sqrt{3} - \mathrm{i}$$

(i) Find the modulus and argument of each of the complex numbers z_1 and z_2 [5]

(ii) Verify that $|z_1 z_2| = |z_1||z_2|$ [5]

(b) The equation

$$z^3 + z^2 + 11z + 51 = 0$$

has a root $(1 - 4i)$.

Find the other two roots of the equation. [6]

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5 Three lines are defined by

L_1 $\mathbf{r} = (-\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

L_2 $\mathbf{r} = (-2\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) + \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$

L_3 $\dfrac{x-3}{2} = \dfrac{y-1}{-1} = \dfrac{z+2}{3}$

(i) Show that the lines L_1 and L_2 are skew. [6]

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(ii) Find the coordinates of the point at which L_2 and L_3 intersect. [5]

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6 **(a)** The complex number z is such that

$$|z| = 6 \qquad \text{and} \qquad \arg z = -\frac{\pi}{3}$$

Express z in the form $a + b\text{i}$ where a and b are real numbers. [4]

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(b) (i) Sketch on an Argand diagram the locus of those points representing complex numbers z which satisfy

$$|z - (3 - 9\mathrm{i})| = \sqrt{10}$$ [3]

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(ii) Hence, or otherwise, find the maximum value of $|z|$ for the complex numbers z which satisfy

$$|z - (3 - 9\mathrm{i})| = \sqrt{10}$$ [4]

7 A matrix **N** is given by

$$\mathbf{N} = \begin{pmatrix} p & 1 & p+2 \\ 3 & -1 & 0 \\ 2 & 1 & 4 \end{pmatrix}$$

(i) Find the value of p for which this matrix is singular. [5]

(ii) If $p = -1$ find, without the use of a calculator, the inverse of **N** [5]

(iii) Hence solve the following system of linear equations.

$$\begin{aligned} -x + y + z &= -3 \\ 3x - y &= 14 \\ 2x + y + 4z &= 18 \end{aligned}$$

[4]

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(iv) If $p = 2$, give a geometrical interpretation for the system of equations

$$\mathbf{N}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 14 \\ 18 \end{pmatrix}$$

[3]

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8 The tetrahedron WXYZ has vertices (1, 2, 3), (–1, 0, 4), (2, 2, 2) and (3, –2, 5) respectively.

(i) Find a Cartesian equation of the plane WXY. [9]

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(ii) Find the volume of the tetrahedron. [6]

(iii) Calculate the acute angle between the line WZ and the plane WXY. [5]

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For Examiner's use only	
Question Number	**Marks**
1	
2	
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Total Marks	

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